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SUBTRACTION BY THE ADDITION PROCESS[1]

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On May 27, 1912, a proposed new course of study in arithmetic was submitted to the district superintendents, who were requested by Dr. Maxwell, then City Superintendent of Schools, to call conferences of principals for the consideration of the course and to report the conclusions of such conferences to him.

Among the innovations proposed by the new syllabus was "subtraction by the addition process." Twenty-five principals were present at my conference. One of these voted in favor of this new method of subtraction. The remaining twenty-four were emphatic in their opposition to it. In our report we said that "the compulsory and exclusive use of the Austrian method of subtraction is disapproved. . . . . The efficacy of methods of subtraction should be experimentally tested before one is made compulsory and all others excluded."

About a year and a half later, the tentative course having been in use meanwhile, Dr. Maxwell again, under date of December 11, 1913, requested district superintendents to report on the arithmetic syllabus after ascertaining the wishes and opinion of principals. My conference reiterated all its previous recommendations, including the suggestion "that the Austrian method of subtraction be not taught exclusively."

The Board of Superintendents, however, thought it best to adhere to its requirement that subtraction be taught "by the addition process," and this prescription has now been in effect for more than six years.

DOES IT FUNCTION?

It is therefore pertinent to ask: "(*a*) What has been the effect of this method of subtraction? (*b*) Is it better than the other methods? (*c*) If so, why? (*d*) Do the children after six years of practice really use the method in their ordinary computation?"

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[1] This communication was recently addressed by the writer to the New York Board of Superintendents. It is being studied by a committee of the Principals Association of the City of New York.

On the first three questions I have no information, although I think it very desirable to have them answered. A scientific investigation is certainly in order, for if the method is superior to others, it should be continued and an effort should be made to have it taught effectually.

The fourth question I am ready to answer so far as Districts *one* and *seven* are concerned.

THE TEST

The principal of each school received copies of the following printed blank with the request that the teachers of the *fourth*, *fifth*, and *sixth* grades be asked to make returns:

SUBTRACTION

P.S. . . . . . . Manhattan Date. . . . . . . Grade. . . . . . . Teacher. . . . .

Directions

At your earliest opportunity, without any explanation, put the following problem on the board and then ask the class the three questions. Speak slowly and repeat the question. After each question, let the children who reply stand. Put down the score. Then return the paper at once to the principal.

In doing this problem: $\begin{array}{r} 61 \\ -29 \\ \hline \end{array}$

	Score
a) How many say: 9 from 11 = 2; 2 from 5 = 3?	. . . .
b) How many say: 9 from 11 = 2; 3 from 6 = 3?	. . . .
c) How many say: 9 and 2 = 11; 3 and 3 = 6?	. . . .

As the Austrian method of subtraction has been compulsory for more than six years, children in these grades have presumably been subtracting by this method for four, five, and six years respectively.

THE RESULT

What does the test reveal? Here are the results in brief:

TABLE I

GENERAL SUMMARY

METHOD	NUMBER OF CHILDREN	PERCENTAGE
a)	2,483	21.8
b)	4,625	40.6
c)	4,260	37.6

Of the 11,368 children registered in these three grades, 2,483, or 21.8 per cent, are using the so-called "first Italian" method of borrowing; 4,625 children, or 40.6 per cent, use the so-called "second Italian" method of borrowing and then adding one to the next figure in the subtrahend; and 4,260 children, or 37.6 per cent, use the Austrian method. That is, only a little more than a third of the children employ the only method which has official sanction. The remaining two-thirds use one of the forbidden methods.

TABLE II
SUMMARY BY GRADES

YEAR	PERCENTAGES		
	(a)	(b)	(c)
4th	13.9	33.4	52.7
5th	22.4	41.2	36.4
6th	31.6	47.2	21.2

In Table II we have additional evidence of the failure of the Austrian method. The proportion of children employing this process regularly declines as we ascend the grades from 52.7 per cent in the fourth grade to 21.2 per cent in the sixth grade. Nor do these figures tell the whole story of the failure. Only 767 children of the 3,621 registered in the sixth grade subtract by the method of addition, and of these 767, 359 are reported by a single school. The percentage of the remaining twenty-six schools is only about 12. In other words, by the time children reach the sixth grade, 88 out of every 100 subtract by a method which is officially excluded from the schools.

WHY THE AUSTRIAN METHOD?

In view of the facts cited, would it not be wise to make a city-wide inquiry to ascertain whether in other districts there is similar failure of the method?

The following arguments should also be answered if the Austrian method is to remain the only authorized way of teaching subtraction:

1. *Psychological.*—The advocates of the process assume that addition and subtraction are psychologically identical, for the stock argument employed in its support is that "it avoids the neces-

sity of learning two separate tables." [Brown and Coffman.] But a slight analysis will show that the mental processes of addition and subtraction are not the same.

Says Professor Judd in *Genetic Psychology for Teachers:*

"Take the simple process of addition, $7+5=12$. . . . . Let us try to express this equation in other terms. We may do so as follows: Here are certain objects; we may arrange them in two unlike groups, and we may put them all together in one group. . . . . The identity consists in the fact that the objects dealt with are the same, and the difference appears in the mode of arranging the objects." [p. 269.]

"Take the process of subtraction as expressed in the equation, $12-7=5$. . . . . The fundamental operation of grouping is here approached from a somewhat different point of view than that adopted in the equation of addition. This equation of subtraction calls attention to the fact that given a group of twelve objects, we may break up the given group into two lesser groups, one of which is specified as seven. After seven are withdrawn from the larger group of twelve, a group of five still remains." [p. 271.]

Dewey and McLellan in *Psychology of Number* make a similar distinction:

"Addition," they say, "may be considered as the operation of finding the quantity which, as a whole, is made up of two or more given quantities as parts, [p 195]. . . . . Addition and subtraction are inverse operations. The one implies the other. . . . . Subtraction in actual operations with objects would seem logically to precede addition. . . . . From what has been shown as to the logical and psychological relation between addition and subtraction, it appears that subtraction is the operation of finding the part of a given quantity which remains after a given part of the quantity has been taken away." [p. 200.]

2. *Practical.*—It has just been shown that in one supervisory district of New York the Austrian method of subtraction is not functioning. It would probably be safe to assume that a like inquiry in other districts would produce similar results. There are several practical reasons that explain the failure:

a) Principals and teachers in the past have been using other methods.

b) It is very difficult to construct practical problems clearly so as to require subtraction by the process of addition. For instance, if John has earned 40 cents and buys a thrift stamp, the common-sense way of dealing with the situation is to ask how much money John has left. The argument about making change does not apply unless we are making change, and then the process is not subtraction at all in the psychological sense, but a concrete process of counting money. If a merchant has 125 yards of cloth and sells 25 yards, ordinary folk do not use the method of making change to find out how many yards are left. If John is 12 years old and Mary 6, the difference of their ages does not suggest making change; nor do we say, "In how many years will Mary be as old as John?"

c) In the problem $\begin{array}{r}764\\ -276\\ \hline\end{array}$ we are advised by Brown and Coffman to say: 6 and 8 = 14; 8 and 7 = 15; 3 and 4 = 7. But suppose a child should ask you to tell him why you do it that way. You would either have to refuse an explanation or resort to one which is beyond the comprehension of children eight years old.